Unemployment and the Theory of International Trade

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I. Introduction

Since Ricardo (1817) and Heckscher and Ohlin (1933) introduced their models in the theory of international trade, the full employment of factor endowments and completely inelastic factor supplies have been often assumed. The assumption is an obviously convenient one. Unemployed workers always exist in the real world. Recent theoretical developments integrate variable labor supply and the theory of international trade. In the framework of Heckscher - Ohlin model, Kemp and Jones (1962) discuss a trade-off between supply of labor and leisure. Thompson (1989) includes variable employment in which the unemployment rate varies inversely with the national income. Davidson and Matusz (2010) combined a job search and unemployment of labor.

In the present study, Thompson's results (1989) are reexamined. In his paper the rate of unemployment (u) is related with the national income (I) by the equation; $du = -\alpha dI$, whereas this relation is modified as $u = 1 - \alpha I$ in this paper. Since α is assumed to be constant, both postulations are the same. However not only our postulation makes analysis easier, but also some ambiguous signs of comparative static result become plain. Nonetheless effects of changing prices on outputs stay ambiguous.

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Specifically the perverse relationship between a commodity price and its output, an increase in the price of a commodity may give rise to a reduction in its output, need to be examined. The paper is concluded with examining whether the equilibrium is stable or not under the variable labor supply model. It turns out that the equilibrium is always stable.

II. Model

We consider a small open economy produces two goods (X and Y) by using two factors of production, say labor and capital. The usual Heckscher - Ohlin assumptions are employed except for the labor market, that is, we allow the existence of unemployment into the model. Let u denote the rate of unemployment.

The employment conditions for both factors are given by;

$$a_{NX}X + a_{NY}Y = (1-u)L \equiv N, \quad a_{KX}X + a_{KY}Y = K. \tag{1-1}$$

a_{ij} denote the quantity of factor i required to produce one unit of good j, K and L respectively denote the endowment of capital and labor, and N denotes total number of labor employed in both sectors.

We assume that the rate of unemployment, u, is related to the national income (I) by the following equation;

$$u = 1 - \alpha I, \quad \alpha = \frac{1}{I_0}, \quad I = wN + rK. \tag{1-2}$$

I_0 denotes the national income when both factors are fully employed and w and r respectively denote the wage rate and the rental rate. The rate of unemployment is inversely related the national income. The idea is similar to Okun's Law in which the change in the rate of unemployment is inversely related to the rate of change in the national income.

Let P_j be the price of a good j, zero profit conditions are given as;

$$P_X = wa_{LX} + ra_{KX}, \quad P_Y = wa_{LY} + ra_{KY}. \tag{1-3}$$

Expressing the above equations in terms of rate of change, we have;

$$\lambda_{NX}\hat{X} + \lambda_{NY}\hat{Y} = \hat{N} + \delta_N\hat{q}, \tag{1-4a}$$

$$\lambda_{KX}\hat{X} + \lambda_{KY}\hat{Y} = \hat{K} - \delta_K\hat{q}, \tag{1-4b}$$

$$\hat{N} = \hat{I} + \hat{L}, \tag{1-5}$$

$$\hat{P}_X = \theta_{NX}\hat{w} + \theta_{KX}\hat{r}, \tag{1-6a}$$

$$\hat{P}_Y = \theta_{NY}\hat{w} + \theta_{KY}\hat{r}. \tag{1-6b}$$

λ_{ij} is the fraction of the ith factor used in the jth production, and θ_{ij} signifies the ith cost share of the jth industry, and where $\hat{q} = \hat{w} - \hat{r}$ and $\delta_K \equiv \lambda_{KX}\theta_{NX}\sigma_X + \lambda_{KY}\theta_{NY}\sigma_Y$, $\delta_N \equiv \lambda_{NX}\theta_{NX}\sigma_X + \lambda_{KY}\theta_{NY}\sigma_Y$ in which σ_j denote the Hicks-Allen elasticity of factor substitution of the jth industry.

Also, expressing the national income (I) in terms of rate of change, we have;

$$\hat{I} = \frac{\theta_{NI}}{\theta_{KI}}(\hat{L} + \hat{w}) + (\hat{K} + \hat{r}), \quad \theta_{NI} = \frac{wN}{I}, \quad \theta_{KI} = \frac{rK}{I}. \tag{1-7}$$

θ_{NI} and θ_{KI} are an income share of employed workers and an income share of capital respectively. Substituting (1-7) into (1-5) yields;

$$\hat{N} = \frac{1}{\theta_{KI}}(\hat{L} + \theta_{NI}\hat{w}) + (\hat{K} + \hat{r}). \tag{1-8}$$

Solving (1-6) for $\hat{w}$ and $\hat{r}$, we obtain;

$$\hat{w} = \frac{1}{|\theta|}(\theta_{KY}\hat{P}_X - \theta_{KX}\hat{P}_Y), \quad \hat{r} = \frac{-1}{|\theta|}(\theta_{NY}\hat{P}_X - \theta_{NX}\hat{P}_Y) \tag{1-9a}$$

$$\hat{q} = \frac{1}{|\theta|}(\hat{P}_X - \hat{P}_Y), \quad |\theta| = \theta_{NX}\theta_{KY} - \theta_{KX}\theta_{NY}. \tag{1-9b}$$

It is clear from (1-9) that the Stolper - Samuelson result stays intact.

III. Comparative Statics

(1) Change in price:

Suppose that endowments of capital and labor stay constant, i.e. $\hat{K} = \hat{L} = 0$. Using (1-8) and (1-9a), we have;

$$\hat{N} = \frac{1}{\theta_{KI}}(\theta_{XI}\hat{P}_X + \theta_{YI}\hat{P}_Y), \quad \theta_{XI} = \frac{P_X X}{I} \text{ and } \theta_{YI} = \frac{P_Y Y}{I}. \tag{2-1}$$

Substituting (1-9b) and (1-10) into (1-4) and solving for $\hat{X}$ and $\hat{Y}$, we may obtain;

$$\begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix} = \frac{1}{A} \begin{bmatrix} A_1 & A_2 \\ -A_3 & -A_4 \end{bmatrix} \begin{bmatrix} \hat{P}_X \\ \hat{P}_Y \end{bmatrix}. \tag{2-2}$$

$A = \theta_{KI} |\lambda||\theta| > 0$, $|\lambda| = \lambda_{NX}\lambda_{KY} - \lambda_{NY}\lambda_{KX}$. A_1, A_2, A_3 and A_4 are defined as;

$$A_1 = \lambda_{KY}\theta_{XI}|\theta| + \theta_{KI}\delta_2, \quad A_2 = \lambda_{KY}\theta_{YI}|\theta| - \theta_{KI}\delta_2, \tag{2-3a}$$

$$A_3 = \lambda_{KX}\theta_{XI}|\theta| + \theta_{KI}\delta_1, \quad A_4 = \lambda_{KX}\theta_{YI}|\theta| - \theta_{KI}\delta_1. \tag{2-3b}$$

$\delta_1 = \lambda_{KX}\delta_N + \lambda_{NX}\delta_K > 0$ and $\delta_2 = \lambda_{KY}\delta_N + \lambda_{NY}\delta_K > 0$. Now suppose that X industry is capital intensive and Y industry is labor intensive, i.e., $|\theta| < 0$.

From (2-3), we may observe; $A_2 < 0$ and $A_4 < 0$, however the sign of A_1 and A_3 can't be determined. This indeterminacy is due to two effects; one is price effect and the other is Rybczynski effect. The second term of A_i, i= 1,2,3,4 signifies a magnitude for the price change and the first term signifies a magnitude for the change in supply of labor. Suppose that price of capital intensive good X increases, then X industry will increase its production, however this price increase also increases the supply of labor through (2-1), this leads to a decrease in production of a good X and an increase in production of a good Y due to Rybczynski effect.

(2) Change in endowment

Suppose the prices of both goods stay constant. From (1-4) and (1-8), we obtain;

$$\hat{X} = \frac{\lambda_{KY}}{\theta_{KI}|\lambda|}\hat{L} + \hat{K}, \qquad \hat{Y} = \frac{-\lambda_{KX}}{\theta_{KI}|\lambda|}\hat{L} + \hat{K}. \tag{2-4}$$

This result differs from the usual sign patterns of Rybczynski theorem. That is, an increase in capital endowment induces increase in both goods. This occurs because an increase in capital endowment increases the supply of labor through (1-8). Note that effects of changing capital endowment on outputs are ambiguous in Thompson (1989).

(3) Changes in N, I and u

Using (1-8) and (1-9a), we obtain;

$$\hat{N} = \frac{\hat{L}}{\theta_{KI}} + \hat{K} + \frac{1}{\theta_{KI}}(\theta_{XI}\hat{P}_X + \theta_{YI}\hat{P}_Y). \tag{2-5}$$

Thus any increase in an endowment and/or a commodity price induces an increase in supply of labor. Also, using (1-7) and (1-9a), we have;

$$\hat{I} = \frac{\theta_{NI}}{\theta_{KI}}\hat{L} + \hat{K} + \frac{1}{\theta_{KI}}(\theta_{XI}\hat{P}_X + \theta_{YI}\hat{P}_Y). \tag{2-6}$$

Similarly any increase in an endowment and/or a commodity price induces an increase in a national income. A change in unemployment rate is related to a change in a national income as;

$$\hat{u} = -\frac{N}{L-N}\hat{I}. \tag{2-7}$$

Hence any increase in an endowment and/or a commodity price reduces the rate of unemployment.

(4) Production Possibility Frontier

The production possibility frontier will change its shape with any change in endowment and will remain intact with any change in a commodity price when both factors are fully employed. However, with an existence of unemployment of labor, an increase in any commodity price will lead to an increase in supply of labor through an equation (2-5), and will affect the shape of the production possibility frontier. Nevertheless the convexity of the PPF will be intact.

(5) Stability of the Equilibrium

With the usual assumptions of the Heckscher-Ohlin model, an equilibrium is always stable through the Marshallian adjustment process. Then a question arises whether an equilibrium is stable or not with an unemployment of labor.

Under an assumption of a small open economy the demand prices are given by the rest of the world, the supply prices are given in an equation (2-2). Let the demand prices be P^*_X, P^*_Y, and let the supply prices be P_{XS}, P_{YS}. The demand prices are constant and the supply prices depend on production of goods X and Y. Thus the Marshallian adjustment process is defined as;

$$\dot{X} = a\left[\frac{P^*_X}{P_{XS}(X,Y)} - 1\right], \qquad (2\text{-}8)$$
$$\dot{Y} = b\left[\frac{P^*_Y}{P_{YS}(X,Y)} - 1\right].$$

$(\cdot)$ on X and Y denote derivative with respect to time. Coefficients, a and b are positive speed of adjustments. Taking a linear approximation of a system (2-8) around the equilibrium (X^* and Y^*), we have;

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \frac{1}{\delta_K} \begin{bmatrix} A_4 & A_2 \\ -A_3 & -A_1 \end{bmatrix} \begin{bmatrix} X - X^* \\ Y - Y_* \end{bmatrix}. \tag{2-9}$$

The equilibrium is Marshallian stable if a trace ($A_4 - A_1$) is negative and a determinant ($-A_1A_4 + A_2A_3$) is positive of the above system. They are obtained as;

$$\begin{aligned} & A_4 - A_1 = -\left[\frac{\theta_{XI}\theta_{YI}(|\theta|)^2}{\theta_{KI}} + \theta_{KI}\delta \right] < 0, \\ & -A_1A_4 + A_2A_3 = \theta_{KI}\delta_K \, |\lambda| |\theta| > 0. \end{aligned} \tag{2-10}$$

Thus the equilibrium is always Marshallian stable.

IV. Conclusion

In this paper some of ambiguous comparative results become plain by using a modified form of the Thompson's equation. That is, an increase in capital endowment leads to an increase in both outputs. The perverse commodity prices-outputs relations are resulted from two effects; price effect and the Rybczynski effectt. It is clear from the equation in (1-9), the Stolper - Samuelson Theorem will not be affected by the specification of the model. Also, in spite of the existence of unemployed workers, the equilibrium is always stable under the Marshallian adjustment process.

References

1. Ball, L. M., D. Leigh and P. Loungani, "Okunslaw: Fit At Fifty?,"
NBER Working paper Series, 18668, January 2013.
2. Davidson, C. and S. J. Matusz, International Trade with Equilibrium Unemployment, Princeton University Press, 2010.
3. Kemp, M. C. and R. W. Jones, "Variable Labor Supply and the Theory of International Trade," Journal of Political Economy, February 1962, 30-36.
4. H. Thompson, "Variable Employment and Income in General Equilibrium," Southern Economic Journal, January 1989, 55, 679-683.