New Approach To Prove Theorems in Heckscher-Ohlin Model

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1. Introduction

The beginning of the international trade course starts with Ricardo model and then Heckscher-Ohlin model. Ricardo (1817) introduced the Theory of Comparative Advantage in the framework of 2 countries, 2 goods and 1 input. More than a century later Heckscher-Ohlin model was introduced in 1933. In 2 goods, 2 factors and 2 countries model (2×2×2 model), they prove that *a country has a comparative advantage in the good that is relatively intensive in the country's relatively abundant factor*. In this 2×2×2 framework, Wolfgang Stolper and Paul Samuelson (1941) obtained a link between international trade and the domestic distribution of income. Known as the Stolper-Samuelson Theorem, it states that a*n increase in the relative price of the labor-intensive good will increase the wage rate relative to both commodity prices and reduce the rent relative to both commodity prices.* In the same framework, T. M. Rybczynski (1955) proved a proposition relating trade and economic growth. Known as the Rybczynski Theorem, it states that *at constant prices, an increase in one factor endowment will increase by a greater proportion the output of the good intensive in that factor and will reduce the output of the other good*. Those three theorems played an important role learning the international trade, and they were

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collected in very influential paper by R. Jones (1965), as an algebraic treatment. To prove those theorems Jones used "hat" notation, i.e., $dX/X = \hat{X}$, a rate of change in X. By the nature of a differentiation, the above theorems are discussed only locally around the equilibrium.

Jones paper is usually taught at the advanced class. Also, those theorems can be almost proved by using diagrams with help of simple equations. This method is usually used for the beginning course, and it is not enough to provide complete proofs. Therefore, it might be useful to provide the proofs without using "a differentiation", or a diagram. Also, the proofs of this paper can be taught at the beginning class., in which we need not limit ourselves to a small change.

We begin introducing a model and assumptions in section 2. In the next sections, we provide the different approach to prove the Heckscher-Ohlin Theorem in section 3, the Stolper-Samuelson Theorem in section 4 and the Rybczynski Theorem in section 5. Conclusion is provided in section 6.

2. Model and Assumptions

The framework of Heckscher-Ohlin model consists of two countries, two goods (X and Y), and two factors of production (call them labor and capital). The assumptions in Heckscher-Ohlin Model are familiar. Thus we briefly state:

(i) The full employment conditions of both factors are given by:

$$a_{LX}X + a_{LY}Y = L, \qquad a_{KX}X + a_{KY}Y = K, \tag{2-1}$$

where a_{ij} denote the quantity of factor i required to produce one unit of good j, and L and K, respectively denote the endowments of labor and capital.

(ii) Perfect competition in factor markets and good markets imply:

$$P_X = wa_{LX} + ra_{KX}, \qquad P_Y = wa_{LY} + ra_{KY}, \tag{2-2}$$

where P_X and P_Y, respectively denote the prices of a good X and a good Y, and where w and r, respectively denote the wage rate and the rental rate.

(iii) The two countries have the same constant returns to scale technology for producing both goods, and also they have identical homothetic preferences.

(iv) There is no factor intensity reversal.

(v) Factors are immobile between countries.

3. The Heckscher-Ohlin Theorem

The Heckscher-Ohlin theorem explains the pattern of comparative advantage in terms of factor endowments. Once the countries engage in free trade and continue to produce both goods, relative commodity price will be equalized between the countries, hence relative factor price will be equalized too (Factor Price Equalization). Also, a_{ij} will be the same between two countries. That is both countries share the common technology matrix A, i.e., A = A* where (*) signifies a foreign country. We can write (2-1) as:

$$A\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} L \\ K \end{bmatrix}, \quad A^*\begin{bmatrix} X^* \\ Y^* \end{bmatrix} = \begin{bmatrix} L^* \\ K^* \end{bmatrix}, \quad A = \begin{bmatrix} a_{LX} & a_{LY} \\ a_{KX} & a_{KY} \end{bmatrix}. \tag{3-1}$$

Since A = A*, we may express (3-1) as:

$$A\begin{bmatrix} X_W \\ Y_W \end{bmatrix} = \begin{bmatrix} L_W \\ K_W \end{bmatrix}, \tag{3-2}$$

where $X_W = X + X^*$, $Y_W = Y + Y^*$, $L_W = L + L^*$, and $K_W = K + K^*$.

Let D_X and D_Y, respectively denote quantity demanded for a good X and a good Y for a home country. Let I and I*, respectively denote a domestic and a foreign income in terms of a good Y, and $P = P_X/P_Y$. Since demand is assumed to be identical and homothetic for both countries, with free trade equalizing relative price we have:

$$\begin{bmatrix} D_X \\ D_Y \end{bmatrix} = s\begin{bmatrix} X_W \\ Y_W \end{bmatrix}, \qquad s = \frac{I}{I + I^*} = \frac{I}{I_W}, \qquad I = PX + Y, I^* = PX^* + Y^*, \tag{3-3}$$

where “s” is home country’s share of world spending. [To obtain (3-3), see Appendix]. Pre-multiply (3-3) by A, and using (3-2) we obtain,

$$A\begin{bmatrix} D_X \\ D_Y \end{bmatrix} = sA\begin{bmatrix} X_W \\ Y_Y \end{bmatrix} = s\begin{bmatrix} L_W \\ K_W \end{bmatrix}. \tag{3-4}$$

From (3-1) and (3-4), we have:

$$A\begin{bmatrix} D_X - X \\ D_Y - Y \end{bmatrix} = s\begin{bmatrix} L_W \\ K_W \end{bmatrix} - \begin{bmatrix} L \\ K \end{bmatrix} = \begin{bmatrix} sL^* - (1-s)L \\ sK^* - (1-s)K \end{bmatrix}. \tag{3-5}$$

Multiply (3-5) by (-1). Then (3-5) can be written as:

$$A\begin{bmatrix} X - D_X \\ Y - D_Y \end{bmatrix} = \begin{bmatrix} L \\ K \end{bmatrix} - s\begin{bmatrix} L_W \\ K_W \end{bmatrix}. \tag{3-5)'}$$

In the Heckscher-Ohlin-Vanek Model, the left hand side of (3-5)' is called "the measured factor content of trade," and the right hand side is called "the predicted factor content of trade". (See Davis 2003).

Let $k = K/L$ and $k^* = K^*/L^*$, respectively denote a domestic and a foreign capital endowment per labor. We may compute the last term of (3-5) as:

$$sL^* - (1-s)L = \frac{rLL^*(k-k^*)}{I_W}, \qquad sK^* - (1-s)K = \frac{wLL^*(k^*-k)}{I_W} \quad . \tag{3-6}$$

Substituting (3-6) into (3-5) and solving for $(D_X - X)$ and $(D_Y - Y)$, we obtain:

$$\begin{bmatrix} D_X - X \\ D_Y - Y \end{bmatrix} = \frac{LL^*(k-k^*)}{|A|\, I_W}\begin{bmatrix} a_{KY} & -a_{LY} \\ -a_{KX} & a_{LX} \end{bmatrix}\begin{bmatrix} r \\ -w \end{bmatrix}$$

$$= \frac{LL^*(k-k^*)}{|A|\, I_W}\begin{bmatrix} P_Y \\ -P_X \end{bmatrix}, \qquad |A| = a_{LX}\, a_{LY}(k_Y - k_X), \tag{3-7}$$

where $k_j = a_{Kj} / a_{Lj}$, $j = X, Y$, are factor intensity in an industry j.

Suppose that a home country is relatively capital abundant ($k > k^*$) and a good X is capital intensive ($k_X > k_Y$), i.e., $|A|<0$, then a home country will export a good X (i.e., $D_X - X < 0$) and will import a good Y (i.e., $D_Y - Y > 0$).

4. The Stolper-Samuelson Theorem

The Stolper-Samuelson theorem links international trade to the domestic distribution of income. We first obtain the relation between P and q = w/r. Let $P_0 = P_{X0}/P_{Y0}$ and $q_0 = w_0/r_0$ are defined by the following unit cost equations:

$$\frac{P_{X0}}{r_0} = q_0 a_{LX} + a_{KX}, \qquad \frac{P_{Y0}}{r_0} = q_0 a_{LY} + a_{KY}. \tag{4-1}$$

By suitable choice of unit, we can set $P_{X0} = P_{Y0}$, i.e., $P_0 = 1$ due to linearly homogeneous technology. Suppose q changes from q_0 to $q_1 = w_1 / r_1$, and suppose $P_1 = P_{X1} / P_{Y1}$ is defined by:

$$\frac{P_{X1}}{r_1} = q_1 a_{LX} + a_{KX}, \qquad \frac{P_{Y1}}{r_1} = q_1 a_{LY} + a_{KY}. \tag{4-2}$$

Let (a^*_{LX}, a^*_{KX}) and (a^*_{LY}, a^*_{KY}) be defined by the intersections of the unit cost equations given in (4-1) and (4-2). (a^*_{LX}, a^*_{KX}) and (a^*_{LY}, a^*_{KY}) can be obtained as:

$$(q_1 - q_0)a^*_{LX} = \frac{P_{X1}}{r_1} - \frac{P_{X0}}{r_0}, \qquad (q_1 - q_0)a^*_{LY} = \frac{P_{Y1}}{r_1} - \frac{P_{Y0}}{r_0}, \tag{4-3a}$$

$$a^*_{KX} = \frac{P_{X0}}{r_0} - q_0 a^*_{LX}, \qquad a^*_{KY} = \frac{P_{Y0}}{r_0} - q_0 a^*_{LY}. \tag{4-3b}$$

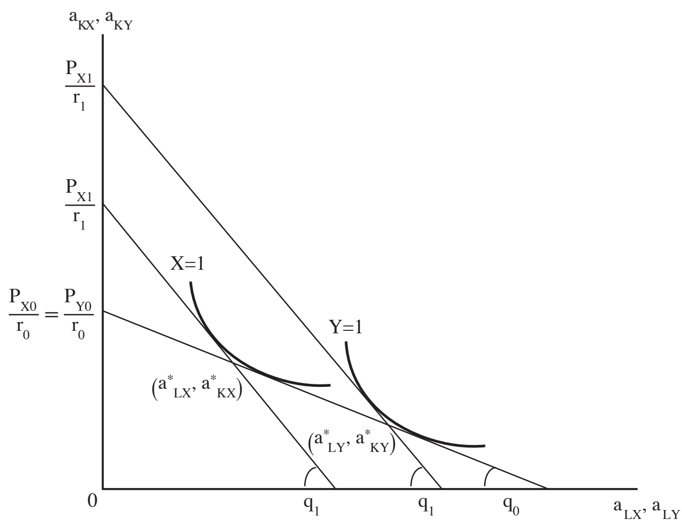
Since $P_{X0} = P_{Y0}$, we may obtain the following relation from (4-3b):

$$k_X^*\left(=\frac{a_{KX}^*}{a_{LX}^*}\right)-k_Y^*\left(=\frac{a_{KY}^*}{a_{LY}^*}\right)=b_0(a_{LY}^*-a_{LX}^*),\qquad b_0\equiv\frac{P_{X0}}{r_0a_{LX}^*a_{LY}^*}.$$

Thus, we may have $(k_X^*-k_Y^*)>0\Leftrightarrow(a_{LY}^*-a_{LX}^*)>0$. How a change in P and a change in q are related is obtained from (4-3a) as:

$$\frac{P_{Y1}}{r_1}(P_1-P_0)=(q_1-q_0)(a_{LX}^*-a_{LY}^*)=\frac{1}{b_0}(q_1-q_0)(k_Y^*-k_X^*).\tag{4-4}$$

Note that $P_0=1$. Suppose $k_x>k_Y$. Due to the assumption (iv) we have $k_X^*>k_Y^*$. Thus $(P_1-P_0)<0\Leftrightarrow(q_1-q_0)>0$. In (4-4) a change in P and q need not be small.



We are now ready to obtain the expressions for Stolper-Samuelson Theorem. Using (4-3a) and (4-4), we obtain:

$$\frac{P_{X1}}{r_1}-\frac{P_{X0}}{r_0}=\frac{a^*_{LX}(P_1-P_0)b_1}{(k^*_Y-k^*_X)},\qquad \frac{P_{Y1}}{r_1}-\frac{P_{Y0}}{r_0}=\frac{a^*_{LY}(P_1-P_0)b_1}{(k^*_Y-k^*_X)}. \tag{4-5}$$

Where b_1 is defined as $b_1 \equiv \frac{P_{Y1}b_0}{r_1}$.

Similarly, we may compute:

$$\frac{P_{X1}}{w_1}-\frac{P_{X0}}{w_0}=-\frac{a^*_{KX}(P_1-P_0)b_2}{(k^*_Y-k^*_X)},\qquad \frac{P_{Y1}}{w_1}-\frac{P_{Y0}}{w_0}=-\frac{a^*_{KY}(P_1-P_0)b_2}{(k^*_Y-k^*_X)}. \tag{4-6}$$

Where b_2 is defined as $b_2 \equiv \frac{b_1}{q_0 q_1}$.

Suppose that $k_x > k_Y$ and P_X increases (i.e., $P_1 - P_0 > 0$). From (4-5) and (4-6), we may obtain:

$$\frac{P_{X1}}{r_1}<\frac{P_{X0}}{r_0},\quad \frac{P_{Y1}}{r_1}<\frac{P_{Y0}}{r_0},\quad \frac{P_{X1}}{w_1}>\frac{P_{X0}}{w_0},\quad \frac{P_{Y1}}{w_1}>\frac{P_{Y0}}{w_0},$$

or

$$\frac{r_1}{P_{x1}}>\frac{r_0}{P_{X0}},\quad \frac{r_1}{P_{Y1}}>\frac{r_0}{P_{Y0}},\quad \frac{w_1}{P_{x1}}<\frac{w_0}{P_{X0}},\quad \frac{w_1}{P_{Y1}}<\frac{w_0}{P_{Y0}}.$$

Hence an increase in the relative price of the capital intensive good will increase the rent relative to both commodity prices and reduce the wage rate relative to both commodity prices.

5. The Rybczynski Theorem

The Rybczynski theorem links economic growth and output levels. When prices are constant, a matrix A stays unchanged. Let (X_0, Y_0) and (X_1, Y_1), respectively denote the output level related with the endowment levels (L_0, K_0) and (L_1, K_1). From (2-1) we may obtain:

$$\begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} a_{KY} & -a_{LY} \\ -a_{KX} & a_{LX} \end{bmatrix}\begin{bmatrix} L_0 \\ K_0 \end{bmatrix}, \quad \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} a_{KY} & -a_{LY} \\ -a_{KX} & a_{LX} \end{bmatrix}\begin{bmatrix} L_1 \\ K_1 \end{bmatrix},$$

$$\begin{bmatrix} X_1 - X_0 \\ Y_1 - Y_0 \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} a_{KY} & -a_{LY} \\ -a_{KX} & a_{LX} \end{bmatrix}\begin{bmatrix} L_1 - L_0 \\ K_1 - K_0 \end{bmatrix}. \tag{5-1}$$

Suppose $k_X > k_Y$(i.e., $|A| < 0$), $L_1 = L_0$ and $K_1 > K_0$. From (5-1) we at once obtain:

$$\begin{bmatrix} X_1 - X_0 \\ Y_1 - Y_0 \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} -a_{LY} \\ a_{LX} \end{bmatrix}(K_1 - K_0).$$

Thus we obtain $X_1 > X_0$ and $Y_1 < Y_0$. At constant prices, an increase in one factor endowment will increase the output of the good intensive in that factor and reduce the output of the other good.

6. Conclusion

New approach to prove four major theorems; the Heckscher-Ohlin theorem, the Stolper-Samuelson theorem and the Rybczynski theorem are shown. As this approach does not use "a differentiation", or a diagram, this may be taught in intermediate class in the international trade course between advanced and beginning classes.

Appendix

Here we obtain (3-3). The markets clearing conditions are given by,

$$D_X + D_X^* = X + X^* = X_W, \quad D_Y + D_Y^* = Y + Y^* = Y_W. \tag{A-1}$$

The identical homothetic demand with free trade equalizing relative price implies,

$$\frac{D_X}{D_Y} = \frac{D_X^*}{D_Y^*} = \frac{X_W - D_X}{Y_W - D_Y} \Rightarrow \quad D_Y = \frac{Y_W}{X_W} D_X. \tag{A-2}$$

Substituting the above D_Y into the budget constrain ($PD_X + D_Y = I$), and solving for D_X and D_Y we obtain (3-3).

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