Purchasing Power Parity and Macroeconomic Equilibria:

A Retrospective Consideration

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1. Introduction

John Maynard Keynes criticized purchasing power parity during his whole life. He defined the equilibrium exchange rate as the one in which both domestic and external equilibria were attained. Nowadays purchasing power parity has revived in macroeconomic theory. Obstfeld-Rogoff model is essentially a model of real economy, where the law of one price is always held. The model, however, introduces money and assumes some stickiness of prices. Because of this, current account is not balanced. In the other words, purchasing power parity is no longer an equilibrium exchange rate in the meaning that Keynes said.

2. An Overview of Obstfeld-Rogoff Model

After World War II, purchasing Power Parity was treated as just an implicit back-

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ground of open macroeconomic model. This situation, however, turned in the 1990s. Obstfeld-Rogoff（1995）proposed a dynamic general equilibrium model with monopolistic competition, which came to be called as new open economy macroeconomics.

The model is consisted from firms which sell differentiated goods and governments. Domestic goods were distributed from 0 to *n* and foreign ones from *n* to 1. Domestic representative individuals have utility function like equation (1). Here β is discount rate and *C* is CES-type consumption index like equation (2).

$$U_t^j = \sum_{t=0}^{\infty} \beta^t \left[\log C_t^j + \chi \log \frac{M_t^j}{P_t} - \frac{\kappa}{2} y(j)_t^2 \right] \tag{1}$$

$$\mathrm{C}^j = \left(\int_0^1 c^j(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \theta > 1 \tag{2}$$

Here *i* and *j* are indices of consumption goods and individuals respectively. Elasticity of goods substitution β is constant and positive. The anti-logarithm in equation (1) is real money balance held by an individual at the beginning of the term *t* ＋1. The last term in the bracket is disutility of labor.

Price index corresponding to consumption index is like equation (3). This is attained by minimizing nominal consumption expenditure *Z* under the condition that real consumption is given. Because of this, *PC*＝*Z*.

$$\mathrm{P} = \left(\int_0^1 p(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \tag{3}$$

Because the law of one price is held in the model, domestic price of domestic goods is $p(i)$, domestic price of foreign goods $p^*(j)$, foreign price of domestic goods

$p(i)/e$, domestic price of foreign goods $ep^{*}(j)$.　Here e is nominal exchange rate. Price levels of both countries are like equations (4) and (5).

$$\mathrm{P}=\left[\int_0^n p(i)^{1-\theta}di+\int_n^1(ep^*(i)^{1-\theta}di)\right]^{\frac{1}{1-\theta}} \tag{4}$$

$$\mathrm{P}^*=\left[\int_0^n\left(\frac{p(i)}{e}\right)^{1-\theta}di+\int_n^1 p^*(i)^{1-\theta}di\right]^{\frac{1}{1-\theta}} \tag{5}$$

We can deduce $P=eP^{*}$ from equations (4) and (5).　This means purchasing power parity.

Next we show budget constraint of government.　When seigniorage revenue is all transferred to people, equation (6) is held.

$$G_t=\tau_t+\frac{M_t-M_{t-1}}{P_t} \tag{6}$$

Here G is governmental expenditure and τ is lump sum tax.　If G is zero, srigniorage transfer and lump sum tax revenue are equivalent.

When individuals can make deal of no risk bonds in the international financial market, representative individual's budget constraint is like equation (7).

$$P_tB_{t+1}^j+M_t^j=P_t(1+r_t)B_t^j+M_{t-1}^j+p(j)_ty(j)_t-P_tC_t^j-P_t\tau_t \tag{7}$$

Here B is bond held by an individual at the beginning of the term, which is real value shown by consumption goods.　And r is real interest rate at the point of time.

An individual's demands are from equation (8).

$$C^j(i) = \left(\frac{p(i)}{P}\right)^{-\theta} C^j \tag{8}$$

By summing up all individuals' demands, we can obtain equilibrium equation of goods *j* like equation (9). Here C^W is world consumption of goods *j*.

$$\mathrm{y(i)} = \left(\frac{p(i)}{P}\right)^{-\theta} C^W \tag{9}$$

3. Solving the Model

Representative individual maximizes utility and decides consumption, production, bond holding and money holding under the budget constraint (7).

$$\max_{C_t^j, y(j)_t, B_{t+1}^j, M_t^j} U_t^j = \sum_{t=0}^{\infty} \beta^t \left[\log C_t^j + \chi \log \frac{M_t^j}{P_t} - \frac{\kappa}{2} y(j)_t^2\right] \tag{10}$$

The first order conditions of optimization are from equation (11) to (13).

$$C_{t+1} = \beta(1 + r_{t+1}) C_t \tag{11}$$

$$\frac{M_t}{P_t} \chi C_t \left(\frac{1 + i_{t+1}}{i_{t+1}}\right) \tag{12}$$

$$y_t^{\frac{\theta+1}{\theta}} = \frac{\theta - 1}{\theta\kappa} (C_t^W)^{\frac{1}{\theta}} \frac{1}{C_t} \tag{13}$$

In the deduction of equation (12), we used Fisher's interest rate arbitrage (14).

$$1 + i_{t+1} = (1 + r_{t+1}) \frac{P_{t+1}}{P_t} \tag{14}$$

Transversality condition is like equation (15). Here $R_{t,t+}$ is the reciprocal of 1 plus real interest rate from the term t to $t+T$.

$$\lim_{T\to\infty} R_{t,t+T} \left(B_{t+T+1} + \frac{M_{t+T}}{P_{t+T}} \right) = 0 \tag{15}$$

Now we deduce stable state where consumption and production are equivalent. After here, the bars over characters show endogenous value in the stable state.

From Euler equation (11), we can show that real interest rate is determined by subjective discount rate in the stable state.

$$\bar{r} = \frac{1-\beta}{\beta} \equiv \delta \tag{16}$$

In the stable state, real consumption and real income are equivalent. Equations (17) and (18) follows.

$$\bar{C} = \delta \bar{B} + \frac{\bar{p}\bar{y}}{\bar{P}} \tag{17}$$

$$\bar{C}^* = -\left(\frac{n}{1-n} \right) \delta \bar{B} + \frac{\bar{p}^* \bar{y}^*}{\bar{P}^*} \tag{18}$$

Current account is not always balanced even in the stable state. The first terms of the right hand sides of equations (17) and (18) are interest revenue or interest payment from surplus or deficit of current account. The second terms are real revenue values from productions.

From here, we assume current accounts are balanced before a shock. Then we can obtain solutions (19). Here 0 shows the value before the shock.

$$\bar{C}_0 = \bar{C}_0^* = \bar{y}_0 = \bar{y}_0^* = \sqrt{\frac{\theta - 1}{\theta\kappa}} \tag{19}$$

Money demand function is like equation (20).

$$\frac{\overline{M}_0}{\overline{P}_0} = \frac{\overline{M}_0^*}{\overline{P}_0^*} = \frac{\chi(1+\delta)}{\delta}\bar{y}_0 \tag{20}$$

In the model, the prices is determined in the previous term. This is the assumption of price stickiness. Because of this assumption, this term is a short one and the next term is a long one where a new stable state is attained.

As for exchange rate, equations (21) to (23) are held in the model. Here hats show short term rates of change. Hats and bars show long term rates.

$$\hat{C} - \hat{C}^* = \hat{\bar{C}} - \hat{\bar{C}}^* \tag{21}$$

$$\left(\widehat{M} - \widehat{M}^*\right) - \hat{e} = \left(\hat{C} - \hat{C}^*\right) - \delta(\hat{\bar{e}} - \hat{e}) \tag{22}$$

$$\hat{\bar{e}} = \left(\widehat{\overline{M}} - \widehat{\overline{M}}^*\right) - \left(\hat{\bar{C}} - \hat{\bar{C}}^*\right) \tag{23}$$

In the Obstfeld-Rogoff model, purchasing power parity is always held. From these equations, we can know that as for rates of change of exchange rate the short term one and the long term one are the same. Financial policies make exchange rate jump into the long term level immediately.

In the Obstfeld-Rogoff model, money neutrality is not held. Corsetti-Pesenti（1887） showed that when elasticity of goods substitution is unity, money neutrality can be held and current account is always balanced. The reason is very simple. In CES-type consumption index (24), Cobb-Douglas-type one follows when $\theta = 1$.

$$C = \left[n^{\frac{1}{\theta}} (C^d)^{\frac{\theta - 1}{\theta}} + (1 - n)^{\frac{1}{\theta}} \left(C^f\right)^{\frac{\theta - 1}{\theta}} \right]^{\frac{\theta}{\theta - 1}} \tag{24}$$

$$C = (C^d)^n \left(C^f\right)^{1-n} \tag{25}$$

4. Conclusion

The Obstfeld-Rogoff model was the moment of revival of purchasing power parity. However the model assumes monopolistic competition and price setting behavior by firms. Then it is rather natural to assume that the firms use differentiated pricing strategy. In the case, the law of one price is broken. It is very strange that the model holds on purchasing power parity. This shows the special character of the model as a new classical macroeconomics which relies on perfectness of markets.

Anyway, the model does not satisfy Keynes's requirement that an equilibrium exchange rate warrants both domestic and external equilibria at the same time. In the model, current account is not always balanced while "full employment" is always attained.

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